ORTHOGONAL LATIN SQUARES OF CHOI SEOK-JEONG

Sung Sook Kim

Dept. of Applied Mathematics

Paichai University

sskim@pcu.ac.kr

ABSTRACT

A latin square of order n is an $n \times n$ array with entries from a set of numbers arranged in such a way that each number occurs exactly once in each row and exactly once in each column. Two latin squares of the same order are orthogonal latin square if the two latin squares are superimposed, then the n^2 cells contain each pair consisting of a number from the first square and a number from the second. In Europe, Orthogonal Latin squares are the mathematical concepts attributed to Euler. However, an Euler square of order nine was already in existence prior to Euler in Korea. It appeared in the monograph Koo-Soo-Ryak written by Choi Seok-Jeong(1646–1715). He construct a magic square by using two orthogonal latin squares for the first time in the world. In this posterr, we explain Choi's orthogonal latin squares and the history of the Orthogonal Latin squares.

1   Definition of magic square.

A **magic square** of order n is an arrangement of n^2 numbers, usually distinct integers in a square such that the n numbers in all rows, all columns, and both diagonals sum to the same.

2   History of magic square.

Chinese literature dating from as early as 650 BC tells the legend of Lo Shu or "scroll of the river Lo". In ancient China there was a huge flood. The great king Yu (禹) tried to channel the water out to sea where then emerged from the water a turtle with a curious figure/pattern on its shell; circular dots of numbers which were arranged in a three by three grid pattern such that the sum of the numbers in each row, column and diagonal was the same: 15, which is also the number of days in each of the 24 cycles of the Chinese solar year. This pattern, in a certain way, was used by the people in controlling the river.
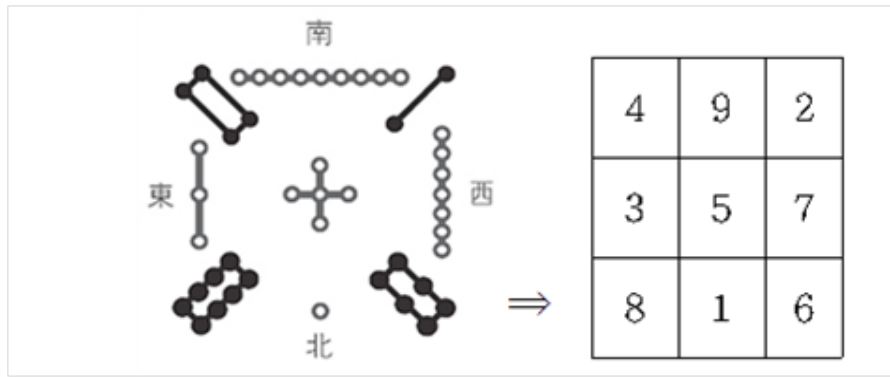
3   Latin square.

A Latin square is an $n \times n$ array filled with n different symbols, each occurring exactly once in each row and exactly once in each column. Here is an example:

2	3	1
1	2	3
3	1	2

1	3	2
3	2	1
2	1	3

4   Orthogonal Latin square.

Orthogonal Latin squares of order n over two sets S and T, each consisting of n symbols, is an $n \times n$ arrangement of cells, each cell containing an ordered pair (s, t), where s is in S and t is in T, such that every row and every column contains each element of S and each element of T exactly once, and that no two cells contain the same ordered pair.

2	3	1
1	2	3
3	1	2

+

1	3	2
3	2	1
2	1	3

=

2, 1	3, 3	1, 2
1, 3	2, 2	3, 1
3, 2	1, 1	2, 3

5   Euler square & Conjecture..

Orthogonal Latin squares were studied in detail by Leonhard Euler, who took the two sets to be $S = \{A, B, C, \ldots\}$, the first n upper-case letters from the Latin alphabet, and $T = \{\alpha, \beta, \gamma, \ldots\}$, the first n lower-case letters from the Greek alphabet—hence the name orthogonal Latin square.

In the 1780s Euler demonstrated methods for constructing orthogona-Latin squares where n is odd or a multiple of 4. Observing that no order-2 square exists and unable to construct an order-6 square (see thirty-six officers problem), he conjectured that none exist for any oddly even number $n \equiv 2(\mod 4)$. Indeed, the non-existence of order-6 squares was definitely confirmed in 1901 by Gaston Tarry through exhaustive enumeration of all possible arrangements of symbols. However, Euler's conjecture resisted solution for a very long time.

In 1959, R.C. Bose and S. S. Shrikhande constructed some counterexamples (dubbed the Euler spoilers) of order 22 using mathematical insights. Then E. T. Parker found a counterexample of order 10 through computer search on UNIVAC (this was one of the earliest combinatorics problems solved on a digital computer).

In 1960, Parker, Bose, and Shrikhande showed Euler's conjecture to be false for all $n \geq 10$. Thus, Graeco-Latin squares exist for all orders $n \geq 3$ except $n = 6$.

6 Orthogonal Latin square Choi Seok-Jeong

5,1 6,3 4,2	8,7 9,9 7,8	2,4 3,6 1,5
4,3 5,2 6,1	7,9 8,8 9,7	1,6 2,5 3,4
6,2 4,1 5,3	9,8 7,7 8,9	3,5 1,4 2,6
2,7 3,9 1,8	5,4 6,6 4,5	8,1 9,3 7,2
1,9 2,8 3,7	4,6 5,5 6,4	7,3 8,2 9,1
3,8 1,7 2,9	6,5 4,4 5,6	9,2 7,1 8,3
8,4 9,6 7,5	2,1 3,3 1,2	5,7 6,9 4,8
7,6 8,5 9,4	1,3 2,2 3,1	4,9 5,8 6,7
9,5 7,4 8,6	3,2 1,1 2,3	6,8 4,7 5,9

37 48 27	70 81 62	13 24 5
30 38 46	63 71 79	6 14 22
47 28 39	80 61 72	23 4 15
16 27 8	40 51 32	64 75 56
9 17 25	33 41 49	57 65 73
26 7 18	50 31 42	74 55 66
67 78 59	10 21 2	43 54 35
60 68 76	3 11 19	36 44 52
77 58 69	20 1 12	53 34 45

$x(i-1) + j$ Koo-Soo-Ryak (Printed by woodblock)

7 Handbook of Combinatorial Design.

On the page 12 of Handbook of Combinatorial Design, the following statement was included.

> The literature on latin squares goes back at least 300 years to the monograph Koo-Soo-Ryak by Choi Seok-Jeong (1646–1715); he uses orthogonal latin squares of order 9 to construct a magic square and notes that he cannot find orthogonal latin squares of order 10.

This implies that the orthogonal latin squares of Choi Seok-Jeong is at least 67 years earlier than Euler's

REFERENCES

– Academy of Korean studies(1991), Encyclopedia of Korean Heritages and culture.

– Allan Adler, *Magic N-Cubes Form a Free Monoid*, the elctronic journal of combinatorics 4 (1997), #R15

– Charles J. Colbourn , Jeffrey H. Dinitz(Editor)(2006), *Handbook of Combinatorial Designs*, 2nd edition, page 12, Chapman and Hall.

– Choi Seok-Jeong(1646–1715), *Koo-Soo-Ryak*, (M. Hur & H. N. Chung, Trans.), Kyowoosa, 2006.

– Hahn Sang-Geun and Oh Yoon-Yong(1993), Choi Seok-Jeong and his magic squares, *Journal of the Korea Society of Mathematical Education* Series A, Vol. 32, No3, 205–219.

– Kim Y. W. and Kim Y. K. (1982), *History of math in Korea*, Yeol Hwa Dang.

– Song, Hong-Yeop, *Choi's orthogonal latin squares is at least 67 years earlier than Euler's*, A presentation to the 2008 Global KMS Conference, Jeju, Korea.

– Lih. Ko-Wei, (2010), A *remarkable Euler square before Euler*, Mathematics Magazine, 83, 163–167.