GEOMETRY TEXTBOOKS IN NORWAY IN THE FIRST HALF OF THE 19TH CENTURY

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ABSTRACT

Bernt Michael Holmboe(1795–1850) wrote the textbooks in mathematics that were predominantly used in the learned schools in Norway between 1825 and 1860, and he was one of the most influential persons in the development of school mathematics in this period. His way of presenting the subject matter was, however, challenged by his colleague and former mentor, Christopher Hansteen (1784–1873). Holmboe's textbook in geometry came in four editions, 1827, 1833, 1851 and 1856, and they were—with one exception—used in all the learned schools in Norway in this period. Holmboe's presentation of the subject matter was in many ways traditional and Euclidean. Hansteen wrote a geometry textbook in 1835 where he challenged this way of presenting the subject matter, and where he let utilitarian considerations overrule logical deduction and theoretical thinking. Holmboe's textbooks where the first Norwegian textbooks that were used in the learned schools, but we know that textbooks by the Danish mathematics teacher, Hans Christian Linderup (1763–1809) was used in Christiania Learned School prior to Holmboe's.

I will give a presentation of the geometry textbooks by Linderup, Holmboe and Hansteen, and their ways of presenting the subject matter with special focus on fundamental concepts like *point*, *line* and *parallel lines*. I will also try to present these textbooks way to present the *parallel axiom* from the Elements by Euclid. By this I will try to describe how geometry teaching traditionally was done in this period, and how it was challenged.

Keywords: history of mathematics education, textbooks, geometry

1 Introduction

Towards the end of the 18th century, a great effort was done to establish mathematics as a school subject in the higher education in Norway, and a school reform that was introduced around year 1800 lead to proper teaching in mathematics. The mathematical community in Norway at that time was small, and all the participants did necessarily become significant members of the society. This was a time of considerable development in the subject of mathematics, and this also influenced the debate about mathematics education.

Cathedral schools[1] were schools from the medieval time that were connected to cathedrals, and they were meant to give a theological education to future priests. All cathedral schools were turned into *Latin schools*, or *grammar schools*,[2] when the reformation was introduced in Norway in 1539, and it was mandatory for every town to have a one. The new Latin schools, together with the old cathedral

[1] Katedralskoler
[2] latinskoler

schools, constituted the so-called *learned schools*. Most of these Latin schools were, however, of a very poor quality, so in reality, the higher education—preceding the university—in 1814, was only four cathedral schools with a total of 200 pupils, in addition to some that had private tuition. The schools were referred to either by their names like *Stavanger Latin School* or *Christiania Cathedral School*, or by for instance *Christiania Learned School*[3]—Christiania was the name of Oslo approximately 1600–1925. By a governmental decree in 1809, the pupils started at the learned schools at the age of 9–10 years, and the duration was normally eight years consisting of four two-year grades, and each day at school was seven hours—four before noon and three after. The university qualifying examination[4] was arranged by the university. The learned schools gave a classic education, and a higher education in scientific subjects at the same standard as the learned schools could be achieved at the Military Academy. This school admitted pupils from the age of 12–14. Several intermediate schools[5] were established in smaller towns after 1814, and they were learned schools without the upper two-year grade. [Andersen, 1914]

Christopher Hansteen describes the situation of the learned schools in the beginning of the 19th century in a footnote in his textbook in plane geometry [Hansteen, 1835, XVI–XVII]. When Christiania Learned School was reformed in 1800, there were in the upper classes hardly any pupil[6] that with skill could do arithmetical operations with whole numbers. Geometry and other mathematical subjects were probably not even known by their names. When Hansteen, together with six other pupils were discharged from Christiania Learned School in 1802 to start at the university, they knew geometry, plane trigonometry, stereometry,[7] conic sections and equations of some other curved lines, the elementary arithmetics, higher-order equations and their solutions, the main statements of combinatorial and permutational analysis, the sum of series, and some astronomy, mechanics and physics—all this due to a very able teacher.[8] Hansteen concludes this footnote by stating that not only is mathematics a subject for cultural formation in the same manner as other sciences, but it is also an indispensable tool in the study of the nature.

2 The textbook authors

Hans Christian Linderup (1763–1809), a Danish mathematics teacher that wrote a textbook in two volumes in 1799–1803, *«The first grounds for the pure mathematics»*,[9] the second edition was published in 1807 [Dansk biografisk Lexicon, 1899, Linderup, 1807]. Linderup writes in the preface that there are several not insignificant modifications, corrections and amendments from the first to the second edition, and the second edition from 1807 will be used in this presentation. Linderup's textbook was used as textbook in geometry in Christiania Learned School *before* Holmboe's books were published.[10]

Christopher Hansteen (1784–1873) was born in Christiania in Norway. He was first a law stu-

[3]Christiania Lærde Skole
[4]examen artium
[5]middelskoler
[6]Discipel
[7]space geometry
[8]Hansteen dedicates his textbook in plane geometry to his former teacher, professor Søren Rasmusen, whose clear and thorough lectures arouse Hansteen's lust for the mathematical sciences.
[9]De første Grunde af den rene Mathematik
[10]Documentation found in the archives of Oslo Cathedral School; teaching reports signed Søren Rasmusen, dated 1810–1812.

dent in Copenhagen, but became interested in the natural sciences when he met the physicist H. C. Ørsted. He became a teacher in applied mathematics at the university in Christiania in June 1814, and he was professor from March 1816 till he retired of medical reasons in 1861. Hansteen was very productive, and wrote about terrestrial magnetism, northern light, meteorology, astronomy, mechanics, etc. He received international recognition after an expedition to Siberia in 1828–30 to study the geomagnetism. Hansteen moved with his family and servants into the new Observatory in Christiania in 1833.

In 1835, Hansteen wrote a textbook in geometry where he challenged the traditional Euclidean geometry, and he introduced the subject matter in a very «un-Euclidean» way. He refers in the preface to Abraham Kästner (1719–1800), a German who published textbooks in geometry in 1758, –63, –74, –88, and they where translated into Danish. Kästner writes that «*any of the countless number of geometry textbooks possesses so much less value and clarity of geometry, the more they deviate from Euclid's Elements*» and Hansteen continues to quote Kästner—«*one should never publish a textbook before one has used it for several years in lectures, and learned to know its shortcomings*». Hansteen then admits that in his book, he has in some sections deviated considerably not alone from Euclid, but also from all the other textbooks he knows, *and* he has not had time to follow the other rule by Kästner, as he has only been working on this book for about 6 months.

Bernt Michael Holmboe (1795–1850) was born in southern Norway as a son of a vicar of the Church. He is mostly known as being the teacher of Niels Henrik Abel, when he was at Christiania Cathedral School. Bernt Michael Holmboe became a student in 1814, and in 1815 he became Christopher Hansteen's assistant at his astronomical observatory. After completing his exams he worked from 1818 till 1826 as teacher at Christiania Cathedral School, then as a lecturer at the university from 1826 till 1834, and after that as a professor. Holmboe was an influential person in the development of mathematics education in the first half of the 19th century. He was the third person to be appointed professor in mathematics at the new university in Christiania. Holmboe was a mathematician at heart, and still a young man when he started teaching in 1818. It is said that his teaching was more «lively» and enthusiastic than what his students were used to. He gave them exercises and assignments out of the ordinary, and he caught their attention [Christiansen, 2009]. Holmboe wrote—with a few exceptions—all the textbooks in mathematics that were used in the learned schools in Norway between 1825 and approximately 1860.

Holmboe wrote textbooks in Arithmetics, Geometry, Stereometry, Trigonometry and Higher Mathematics. These were the textbooks in mathematics that were predominantly used in the learned schools in Norway between 1825 and 1860, a decade after Holmboe's death. He was probably one of the most influential persons in the development of school mathematics in the first half of the 19th century in Norway. His way of presenting the subject matter was in many ways very traditional, and they were challenged by his colleague and former mentor, Christopher Hansteen.

The table 1 shows an overview over Holmboe's textbooks, their Norwegian titles and their various editions.

An Overview Over Holmboe' s Textbooks

TITLE	Edition	Year	Edited by	Publisher
Lærebog i Mathematiken	1st	1825		Jacob Lehmann
Første Deel, Inneholdende Indledning	2nd	1844		J. Lehmanns Enke
til Mathematiken samt Begyndelses-	3rd	1850		J. Chr. Abelsted
grundenetil Arithmetiken	4th	1855		R. Hviids Enke
	5th	1860		R. Hviids Enke
Lærebog i Mathematiken	1st	1827		Jacob Lehmann
Anden Deel, Inneholdende	2nd	1833		Jacob C. Abelsted
Begyndelsesgrundene til Geometrien	3rd	1851	Jens Odén	R. Hviids Enke
	4th	1857	Jens Odén	J. W. Cappelen
Stereometrie	1st	1833		C. L. Rosbaum
	2nd	1859	C. A. Bjerknes	J. Chr. Abelsted
Plan og sphærisk Trigonometrie	1st	1834		C. L. Rosbaum
Lærebog i den høiere Mathematik	1st	1849		Chr. Grøndahl
Første Deel				

3 The textbooks

3.1 [Linderup, 1807]

[Linderup, 1807] emphasizes a platonic view on the being of mathematics when he in the first section explains the fundamental concepts of *body*, *face*, *line* and *point*. A body has three dimensions, a face has two, a line has one, and a point has no dimensions. The boundary of a body is faces, the boundary of a face is lines and the boundary of a lines is points. The boundary of a «thing» is not a part of the «thing», but on the contrary its cessation—the face is not a part of the body, the line is not a part of the face, and the point is not a part of the line. Innumerable points constitute therefore no line, innumerable lines constitute no faces, and innumerable faces constitute no body. From this one may conclude that it is not possible to draw geometric points, lines and faces, as all drawn objects are real physical bodies, whose physical extension in space are geometric bodies. We must, in other words, *abstract* the concept from drawn bodies. [Linderup, 1807, 145–146]

Linderup then gives an explanation of *lines* by imagining a point—which is without any extension—set in motion. This point runs through a trajectory,[11] or a length, without breadth and thickness, and that is a *line*.[12] If the direction[13] of the point is unchanged during the motion, it describes a *straight line*,[14] or *recta*, and the point moves from one place to another following the shortest trajectory. If, on the other hand, the direction of the point is changed during the motion, then it describes a *curved line*,[15] or *curva*. Linderup admits that these explanations are the simplest ideas we may have on straight and curved lines—«*The concepts of straight and curved lines are so plain that they are not possible to define in an understandable manner*».[16] That is why Linderup turns to this circular explanation by explaining «line» by using «direction», and explaining «direction» by using «line». [Linderup, 1807, 146–147]

Linderup's definition of parallel lines is two straight lines in a plane that, no matter how far they

[11] Vey
[12] Linie
[13] Retning
[14] ret Linie
[15] krum Linie
[16] Begreberne om rette og krumme Linier ere saa enkelte at de ikke lade sig definere eller gjøre forstaaelige ved Forklaring

are prolonged, never will meet [Linderup, 1807, 168–169]. In a following theorem, he proves that two straight lines are parallel if, when intersected by a transversal, either

1. the sum of consecutive interior angles equals two right angles
2. corresponding angles are equal
3. alternate angles are equal

Linderup gives some descriptions on how to use compass, ruler and protractor in spite of his platonic view on geometric objects.

3.2 [Holmboe, 1827, Holmboe, 1833]

Holmboe's textbook in geometry came in a total of four editions, but only the two first were published in Holmboe's lifetime. There are some differences from the first edition to the second, but not concerning the fundamental concepts discussed in this paper.

The textbook in basic geometry [Holmboe, 1827] starts with several definitions of basic concepts. The very first definition describes *geometry* as a science about the *coherent magnitudes*. Coherent magnitudes are the space with all available dimensions and time. According to [Solvang, 2001], Holmboe's way of organizing the subject matter was influenced by Adrien-Marie Legendre's (1752–1833) introduction to geometry [Legendre, 1819]. The geometry of Legendre is constructed mainly the same way as Euclid, and starts with a long list of what he calls *explanations*, similar to what Euclid calls *definitions* [Euclid, 1956].

The first definition in [Legendre, 1819] defines *geometry* as a *science which has for its objects the measure of extension. Extension has three dimensions, length, breadth, and thickness*. With reference to classification of coherent magnitudes in space and time, Holmboe classifies geometry in two parts:

1. *The real geometry* defined by the relations between the various magnitudes in space, without considering their changes in time.
2. *Mechanics*, defined by the changes the magnitudes goes through in time. All changes on a magnitude through time are called motion, and it is conditioned by force.

It is postulated that the space stretches indefinitely.[17] Holmboe advises the teacher to show moderation in the review of proofs, and to show examples using numbers before the examination of the proof. This practical advice contradicts the structure of his textbooks, which is strictly Euclidean. There are few exercises and numerical examples, and the notion of construction means to elucidate the concept, and not to use compass and ruler. Holmboe does not give any detailed instructions on how to use ruler and compass in this book, nor does he mention geometric locus, but writes about elucidative[18] objects, magnitudes and concepts. His idea may be that the mathematics teaching shall educate the students with respect to formal logic, by encouraging them to think and conclude.

[17] Geometrie er en Videnskab om de sammenhængende Størrelser. Sammenhengende Størrelser er ere Rummet med enhver deri forekommende Udstrækning og Tiden. Med Hensyn til de sammenhængende Størrelsers Inddeling i Rum og Tid, inddeles Geometrien i 2 Dele. (1) Den egentlige Geometri, der bestemmer de i Rummet forekommende Størrelsers Forhold til hinanden uden Hensyn til deres Forandring i Tiden. (2) Mekanik, der bestemmer de Forandringer, som Størrelserne undergaae i Tiden. Anm. Enhver Forandring, som en Størrelse i Tiden undergaaer, kalles Bevægelse, hvis betingelse kaldes Kraft. Fordringssætning. Rummet maa tænkes udstrakt i det Uendelige.

[18] anskueliggjørende

Holmboe's first definition is a description of a classification—any bounded part of the space is called a *body*,[19] the boundary of the body is called a *face*,[20] the boundary of a face is called a *line*,[21] and the boundary of a line is called a *point*.[22] A body has three extensions, called *length*,[23] *breadth*[24] and *height*,[25] a face has two extensions, called *length* and *breadth*, a line has one extension, called *length*, and a point has no extensions. Holmboe then states without any explanation, but with an illustration,

> «*Fundamental concept. A straight line.*»[26]

A *curved line*[27] is a line of which no part is a straight line, and a *straight plane*[28] is a plane where one, between two arbitrary points may draw a straight line. The fundamental statements of the straight line is that a straight line may be prolonged infinitely, one may always draw one straight line between two points, and one may never draw more than one straight line between two points. The part of the straight line that lays between the two points is the shortest of all lines drawn between the points, and it is called the *distance*[29] between the points. [Holmboe, 1827, 2–4]

Holmboe has in his textbook two subsections called «*About two straight lines intersected by a transversal*»,[30] and «*About parallel lines*».[31] The first subsection gives a thorough description of all pairs of angles this situation produces. This is followed by the consequences of two corresponding angles being equal, and vice versa, the situations which have the consequence that the corresponding angles are equal [Holmboe, 1827, 11–16]. The latter of the two subsections then has a theorem with proof which states that when two straight lines are intersected by a transversal, such that an outside angle is equal to its corresponding interiour angle, then the two straight lines can not intersect no matter how far they are prolonged in both directions. This is followed by Holmboe's definition of *parallel lines*.

> «*Two straight lines in the same plane that does not intersect when prolonged indefinitely to both sides, are* parallel *to each other, or the one is parallel to the other*»[32] [Holmboe, 1827, 46]

In two following theorems, Holmboe demonstrates, using the same situation of two straight lines intersected by a transversal, that when the two straight lines are parallel, then the corresponding angles are equal, and if one of the angles in a pair of corresponding angles is greater than the other, then the two straight lines are *not* parallel. [Holmboe, 1827, 50–53]

Holmboe is in this textbook very true to the *Elements* [Euclid, 1956], but without ever referring to Euclid.

[19] Legeme
[20] Flade
[21] Linie
[22] Punkt
[23] Længde
[24] Brede
[25] Høide
[26] Grundbegreb. En ret Linie.
[27] krum Linie
[28] ret Flade
[29] Affstanden
[30] Om to rette Linier, som overskjæres af en tredie
[31] Om parallele Linier
[32] To rette Linier i samme Plan, som til begge Sider forlængede i det Uendelige ikke skjære hinanden, siges at være *parallele* med hinanden, eller den ene at være parallel med den anden

3.3 [Hansteen, 1835]

In 1835, Christopher Hansteen published a textbook in basic geometry [Hansteen, 1835], which in many ways challenged Holmboe's textbooks. Hansteen's book was 278 pages, which is a lot more than what is expected of a textbook in elementary geometry. The author is intentionally trying to tear down the walls that existed between the classical geometry on one side, and the newer analytical geometry and the infinitesimal geometry on the other. The basis of the textbook is real life, with references to artifacts like corkscrews, stove pipes and hourglasses. The presentation of the subject matter is very unlike Euclid's Elements. The style is narrative and written in the first person, sometimes very lengthy, and there are many numerical examples. Hansteen tried to expand Euclid's definition of straight lines and of parallel lines, and Euclid's parallel postulate [Euclid, 1956].

Hansteen's textbook contains a comprehensive preface which also contains definitions of fundamental concepts. The first concept to be defined is the straight line [Hansteen, 1835, III–IV], which is also, according to Hansteen, «*the foundation of geometry*».[33] It is of great importance that this concept is clearly defined, especially in a science that demands a consistent and logic practice. Hansteen presents five different ways a straight line may be defined

- «*A straight line is a line which lies evenly with the points on itself*» from [Euclid, 1956]. Close to this is also Baron Wolff's definition stating that «*a line is straight when a part is similar to the whole*».[34]
- Archimedes, and most French geometers after him, defined the straight line as «*the shortest trajectory between two points*».
- Some geometers regard the straight line as a hereditary concept that only needs to be mentioned to be understood, and defines a straight line as «*those things which is known to be a straight line*».[35]
- Abraham Kästner says that «*a straight line is that whose points all bear against one trace*»,[36] and he adds that «*no one will learn the straight line to know from an explanation, and no one needs to; but one may say something about it, that guides the attention to a closer attention to what makes it a straight line*».
- Finally, others say that «*when a point moves continually in the same direction, then its trajectory is a straight line*».

According to Hansteen, after such definitions, all geometers introduces a postulate which states that «*one may create a straight line between two given points, and prolong such a given straight line in any direction in both directions as one pleases*». Hansteen makes noteworthy objections to such a postulate by asking with what tool such a prolonging shall be made, and how to make sure that the line made by such a tool is homogeneous, or that is satisfies the demands made in the various definitions of a straight line [Hansteen, 1835, VI–V].

Hansteen elaborates towards a definition where he let lines be produced by the movement of a point, and there are two kinds. One kind has the quality that when two points of a part of the line are placed on two arbitrary points on the whole line, then all points of the part of the line will coincide with points in the whole line—analogous, if we let a part of a line move along the whole line, and the part always fits with the whole line. Such lines are called *homogeneous lines*,[37] and there are two types—*straight* and *curved* lines. A homogeneous line has all over the same curvature, and all perpendiculars of any plane homogeneous line will, when duly prolonged, either intersect in one

[33] Geometriens Grundvold
[34] *Linae recta est, cujus pars quæcunque est toti similis*
[35] *Qvæ linea recta dicatur notum est*
[36] en ret Linie er den, hvis Punkter alle ligge hen mod een Egn
[37] eensartede Linier

point, or not intersect at all. There are in other words only two types of homogeneous lines in a plane—the straight line and the circle. When a point moves from one place to another in a space, then it describes a line. If this line is straight, it is called the direction of the motion [Hansteen, 1835, IV–V,7,9,35–38] From the concept of the straight line we may derive the concept of the plane, and from these definitions we may prove that a line is straight when all the points in the line remains unchanged in the same position is the line is rotated around two arbitrary points on the line [Hansteen, 1835, 9], and that a straight line between two points is shorter than any curved or broken lines between these two points [Hansteen, 1835, 40].These two statements are not axioms, but theorems.

It is more proper that a «mechanical artist» derives rules for his practice from the definitions and theorems of the geometry, than that the theoretical geometer shall direct his concepts and definitions towards this practice. The carpenter's planer and the metalworker's file are tools that are suitable for producing homogeneous planes and lines, and the geometer should not neglect to acquire the theoretical principles on which these methods are based. A *ruler* is described as a tool—made by wood or metal—by which one may produce straight lines in a plane. [Hansteen, 1835, VIII,13]

The cause for the much discussed controversy Hansteen's textbook made was the handling of parallel lines. Hansteen states very clearly that the Euclidean definition of parallel straight lines, embrace by nearly all geometers, has all the logic errors a definition may have. He states correctly that parallel lines are defined, according to Euclid, by a *negative* quality, and not a *positive*. He continues by stating that the quality by which the parallel lines are defined, is *outside all experience and test*, as it points towards the infinite. Euclid's definition may also not be used on curved lines, which may also be parallel—according to Hansteen. «*No one will hesitate in declaring to two concentric circles reciprocal parallel*». There is a definition stating that if two lines in a plane never intersect, no matter how far they are prolonged in any direction, does not make an angle [Hansteen, 1835, 28]. There is, however, no mentioning that these lines are parallel.

Hansteen argues for an understanding of parallelism where one let a perpendicular to any kind of line move along this line, in such a way that it always is a perpendicular. Any point on this perpendicular then describes a line, where any point's smallest distance to the original line all over is the same [Hansteen, 1835, IX]. Consequently, Hansteen has a definition of parallel lines

> «*Any line that is being described by a point on the perpendicular to a given line, when it moves along the same with an unaltered angle, is said to be parallel to the directrix*» [Hansteen, 1835, 59]

and the characteristics of a line, parallel to another, is therefore

- it always cuts off equal parts of all its perpendiculars
- any perpendicular to one of these lines is also a perpendicular to the other

and parallel of a straight line has in addition the following characteristics

- the parallel is also a straight line
- as these straight lines never intersect, they form no angle with each other
- if the parallel lines are intersected by a transversal, then the alternate interior angles are equal, the corresponding angles are equal, and the consecutive interior angles equals 2R

By following these properties of parallelism, Hansteen transform Euclid's disputed axiom into a theorem which he proves [Hansteen, 1835, 70].

Hansteen does all over let lines and planes be produced by the motion of points and lines, because this method gives the clearest conception of a line's direction in any point. One may easy imagine that a point in motion has a certain bearing in any place of its trajectory. Some geometers object to this method since motion involved *time* and *power*, two concepts that are irrelevant to geometry, but belongs in the mechanics. Hansteen states that the motion of an immaterial point requires now power, and that we are only elucidating a motion in our minds. [Hansteen, 1835, XII]

Hansteen's textbook was published in one edition only, but one reason may be that it contained much subject matter outside the school curriculum. He explains that because of a limited production of textbooks in Norway, he has added subject matter beyond the curriculum of the learned schools, but should be of interest for students that want to prepare themselves for a study of the higher mathematics [Hansteen, 1835, XVIII]. It is also worth while to mention as a curiosity that Hansteen in his textbook introduces and describes *Metre* as a the new unit of length [Hansteen, 1835, 81].

4 A comparison, and some concluding remarks

Hansteen's textbook is a textbook that stands out, and is different from the contemporary textbooks. Both Holmboe's and Linderup's books were firmly in the Euclidean tradition that was typical for geometry textbooks in the 18th and 19th century. There was a present debate about the use of Euclidean ideas in textbooks in geometry, and when Christopher Hansteen published his textbook in geometry [Hansteen, 1835], it was evidently a controversial issue, and an attack on the Euclidean textbooks [Piene, 1937, Solvang, 2001].

It is an noteworthy difference that Hansteen tries to give a thorough definition of *lines*, both straight, curved and broken, whilst both Linderup and Holmboe accept the straight line as a self-evident concept that is understood, and does not need to be explained.

The most interesting difference between Holmboe and Hansteen is their understanding of parallelism. Holmboe's definition of parallel lines is similar to Euclid's, and he was also firmly in agreement with Legendre's understanding of parallelism. Hansteen's definition of parallel lines covers all types of lines—straight and curved, homogeneous and not homogeneous. A bitter controversy broke out between Holmboe and Hansteen, and the polemics that followed in the newspapers has later been called the dispute about parallelism. Both Holmboe and Hansteen published pamphlets where they justified their own views.

Holmboe's textbooks were more or less controlling the market regarding textbooks in mathematics in the first half of the 19th century.

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