CALL TO REVISIT MESOAMERICAN CALENDARS
The One That is Called the Real Calendar

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ABSTRACT

If necessary, we shall review the numerations used by the Mayas and the Aztecs. We will propose a survey of Mesoamericans' arithmetic and time calculations. And, we will present and discuss a main thesis.

The thesis in focus has two major aspects: historical and epistemological. The presentation will take up and criticize the traditional belief that affirms that Mesoamerican peoples shared the same type of calendar, the main characteristics of which are the two following: a) this calendar would have been obtained by the combination of the almanac and of the solar vague year, two cycles respectively of 13×20 days (dated by expressions of the form $\boldsymbol{\alpha X}$) and of $18 \times 20 + 5$ days (which the Mayas dated by 365 expressions of the form $\boldsymbol{\alpha Y}$, and b) this combination should have produced, in the Mayan case: a Calendar Round not of 260 x 365 = 94 900 days but rather of only 18 980 days dated $(\boldsymbol{\alpha X}, \boldsymbol{\beta Y})$, or, in the Aztec case: an Aztec century of 52 years distinguished by expressions of the form $\boldsymbol{\alpha X_P}$. From an epistemological point of view, we will survey the why and how of some historical misunderstandings of the most original creations made by the ancient Amerindians, and the fact that certain colonial documents asserted that the Indians had a 'real' calendar, that is to say a calendar in sync with the annual course of the sun.

In contrast to the thesis b), my second objective is not really a proposition or a conjecture to be demonstrated, but a presentation of some Mayan and Aztec creations in arithmetic and time computations. These creations that could advantageously enter the curriculum of the classes of mathematics to widen educational horizons by teaching students to include, accept and understand the real problem of translation among foreign thoughts and cultures. Certainly, some ancient Mesoamerican creations may be taken up directly by mathematics classes, on the condition that teachers will first be trained in this very uniquely-evolved cultural domain. This, indeed, is a necessary condition for anyone wishing to avoid both the impulsive projections or interpretations which led Caramuel (1670) to produce a 'monstrous hybrid' as called by Hernández Nieto (1978) and which led Waldeck (1838) to see elephants on the text of the central panel of the Temple of the Inscriptions of Palenque.

1 Invitation to look mathematics made outside of the Occidental realm

My presentation invites researchers, teachers and didacticians of mathematics to step outside the disciplinary monologues and to open the windows of the classroom and of students' minds to diversity in the cognitive world.

1.1 Open your mind

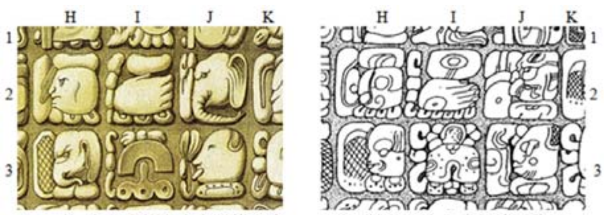
For being mathematically educated is also to be able to escape the routines and to have a free thought, it is also to know how to reject arguments based on authority or faith. This type of education is obtained by exposing one' s self to the problems of others, interactively and in proportion to the openness of each to the questions that call out to men of all times, places, cultures and languages. To more clearly focus on the essential, my presentation is based on the comparative analysis of calendars used in the Ancient Mesoamerica by Mayan and Aztec peoples. This presentation will examine the regards cast upon the mathematical practices that were made and unmade outside of the Occidental realm. It will be shown, in particular, how the Europeans caused, before having even understood it, the loss of the Mesoamerican expertise concerning calendars and computation. It also gives credence to those, who, in the Colonial period, claimed to have and to use a calendar that they qualified as "real", a calendar that did not lie and did not need to be reset. In doing so, we will discover that a wall of incomprehension looms before anyone who engages in a genuinely original or profound thought, even when it is as simple as a child' s expression, a student' s question or a first step into Mayan arithmetic knowledge.

Inviting the creations of Mesoamerican scribes into math class may contribute to the development of the capacity to translate and to break down the walls of incomprehension, and it is also an homage to the memory of the Numbers, the Numerations, the Calendars··· forgotten in the shadows cast by the expansion of the European Enlightenment.

1.2 Shock between Ancient and New World

When a people conquers another, exploits them and imposes its language and its currency, the cultural values of the indigenous peoples react brutally to the slightest decisions of the actors engaged in these historic circumstances. At the beginning of the colonization process, the conquerors deny any valuation of the immaterial productions of the peoples whom they are in the process of vanquishing, overcoming and exploiting[1]. Later, when the atrocities committed begin to be known, voices are raised denouncing all kind of ethnocide. Often in vain. On the one hand because the mass of the colonizers will not heed these voices, and, on the other hand, because the weight of time makes it difficult to see and to admit the existence of ways of thinking which are so radically different.

For this reason, even the most understanding cannot avoid acting in their own self-interest and finish, like the explorer Waldeck, by hallucinating elephants while copying the glyphs from the central panel of the Temple of Inscriptions of Palenque. Here are two fragments of the panel showing the glyphs J2 and H3 respectively drawn by Waldeck and by Schele (an expert in Maya epigraphy):



Jean-Frédéric Waldeck (1766/1875) **Linda Schele** (1942/1998)

Beyond provoking a smile, Waldeck' s approach bears a lesson: sure of the architectural prowess of their pyramids, deserted for centuries, but ignoring virtually everything of the prestigious past of the Mayas who had built them, Waldeck sought to explain their presence and their beauty. A credi-

[1] If pressed, they sometimes finish by explaining their exactions by providing justifications which are as false as they are sectarian and partisan: since we are defeating them, their gods have clearly abandoned them, and even if their gods have fled, it is because their works are the fruit of the devil or of their savage nature. In either case, they must thus be destroyed and eradicated, and their authors must be punished or educated; in short, indoctrinated or reconditioned.

ble civilizing force was thought to be found in the great African civilization of Egypt. In doing such, Waldeck placed himself within the small circle of a handful of scholars and explorers who fought against the *doxa* of the period which affirmed that Mesoamerican cultures were inferior to those of the ancient world. And the star civilization of the period was Egypt, following its revelation to the world through the embarkation of 68 scholars in Bonaparte' s 1799 campaign. In spite of the open mindedness of Waldeck and of a few other scholars, the Mayas (and the Aztecs) were viewed as savages or, at best, as half-civilized. The title, *Notes on the **semi-civilized** nations of Mexico, Yucatan, and Central America* by Albert Gallatin (1845) perfectly illustrates the ambiguous, touching and arrogant nature of these scholars, both sufficiently open to study the facts of indigenous cultures and insufficiently open enough to grant them the title of "civilized" and to cease describing them as children, primitives or irrational beings. During the XIXth century, Mesoamerican values were relegated to the secondary market of semi-values. Well beneath those of the Greek or Roman classics, and even inferior to more prized exotic values such as those of the Egyptians or the Chinese.

The same type of incomprehension of the other has been described by Hernández Nieto (1978) under the appellation of "monstrous hybrids". This concept is clearly exemplified by the study of the works of a Spanish Cistercian of Czech origin, Juan Caramuel de Lobkozitz (1606/1682). Concerned with "penetrating with an open spirit the arithmetic in use in ancient Mexico", Caramuel refuted the erroneous affirmation of Brother Alonso de Molina stating that Aztec numeration did not exceed 8 000[2]. On the other hand, he also rejected the exact and recognized theory of the vigesimal nature of Nahuatl numeration. Regarding this, Caramuel presents his own interpretation of the numeral expressions that he has "manipulated" to the point of perverting the Nahuatl numeration into a purely quinary system[3](base 5 numeration). In other words, Caramuel' s vision, which is both curious and benevolent, produced a hybrid – the purely quinary "Caramuelian" system mixes Aztec numerals and those invented by Caramuel – the parthenogenetic fruit of his solitary manipulation of Nahuatl numeric expressions[4]. Caramuel' s hybrid is monstrous, first because he fuses groups of numeric expressions which were conceived and which develop in worlds having virtually no points in common: a numeration that is both Aztec and quinary simply does not exist. Second, because Caramuel' s chimeras are still-born which never have had any existence outside the study by H. Nieto (1978) of the Caramuel' s manuscript. Otherwise stated, Caramuel allowed himself to be carried away by an "impulsive interpretation" (Luria;1966).

Incomprehension of the thoughts of others obstructs, via the prejudices it provokes, the work of scientists, whether that of the epigraphist or the historian. Diego de Landa, one of the very first framework familiar to him: that of Latin alphabetic writing. Incapable

[2]Caramuel wrote "but I show that the Aztecs arrived at the number of 31 250, or even further" (p. 93). Limited to 160 000 for the mathematician Geneviève Guitel (1975).

[3]"Según el sistema defendido por nuestro autor las operaciones serían todas dentro de un orden quinario, en el cual cinco rayas darían una unidad del orden superior, y cada posición equivaldría a 5^2" (p. 91).

[4]Which constitute – in the opinion of all witlessness' who have testified on the witness stand of History since the beginning of the Conquest – a vigesimal system of numeration of a "well-organized variety" in the terminology of Guitel (1975).

[5]For the (poor) reason of being able to better combat "idolatry" in the Mayan texts.

'monk-ethnologists', provides a perfect illustration. In trying to understand[5] Mayan writing, he was unsuccessful in escaping the only of imagining another type of writing, he attempted to force Mayan writing into the alphabetical mold. For centuries, his "alphabet" blocked epigraphists who finally managed to see that Mayan writing is of the logo-syllabographic type.

Whether called a "monstrous hybrid", "Waldeck's elephant", or even the "self indulgence of the ethno-X", the concept reflects not the ineluctable fact of projecting its own frames of reference and its own forms of knowledge upon the foreign work that it is trying to understand, but rather the failure to submit all readings and interpretations to systematic and collective criticism, criticism that is at the least interdisciplinary and interethnic or intercultural.

Whatever the motivations, in face of the Occidental productions, both material and immaterial,

pushed by the Spanish colonists, occasionally the curiosity and desire of the Autochthones matched that of the Europeans regarding their diffusion and adoption. And vice-versa. With the exception that the inequality which is inherent to the condition of servitude has strongly limited the transfers in the Autochthon-European direction to the simplest forms. For example, the inescapable fact that only the Autochthons are forced to learn the language of the latter. Thus, the Amerindian languages are in danger everywhere, while in every State of the continent the official European languages have been enriched over five centuries by a more or less important number of lexicological and grammatical contributions which, of course, distance them somewhat from the languages of their actual neighbors and ancient metropolises. Today for example, the Spanish spoken in Merida (Yucatan) is not exactly that spoken in Merida (Venezuela). Nor that you hear in Spain or on the *Zocalo* of Mexico.

The elements of history of the numerations presented in this talk show that the "soft" form of colonization is often proved to be more "effective"[6] in the long term than the brutal form of those who seek the pure and simple eradication of the Autochthon's creations and the death sentence for their native creators. As I stated in beginning this introduction, the result of a half-millennium of European colonization is very clear as far as numerations are concerned. The current inhabitants of Mesoamerica all use[7] in their day-to-day lives: both the metric system imposed by the Revolutionaries of 1789 as well as the numeration of decimal position and Indo-Arabic numerals that they exhibit, for example, on the license plates of the vehicles driven today by certain descendants of the Mayas, the Aztecs, and others. Otherwise stated, the meeting between the two worlds unleashed a chain reaction leading to a numerical deculturation – if we may so say – that is nearly complete. A catastrophic evolution (in the modern sense of determinist chaos theories) which leads to the disappearance of the vigesimal numerations of two of the great American cultures: the numeration of position of the

[6]Thus, from a certain point of view, the "soft" is more menacing and dangerous for the indigenous productions. History should not forget that the Republican institution of "public, secular, free and obligatory" schooling, in France at the beginning of the XXth century, had as a collateral effect the disappearance from the public sphere, in only three generations, of the majority of regional languages, of which the most vigorous, like Basque or Breton, owe their survival to the force of will of militants who succeeded in integrating them into the school system and a select few other public spaces such as radio or TV waves.

[7]Even the menus at Chinese restaurants note their prices in Indo-Arabic numerals!

Mayas and the additive numeration of the Aztecs[8].

Like the French (and others!) who still know and use upon occasion the numeration in roman numerals, the Mayas or the Aztecs may still write numbers in the vigesimal numeration of days gone by, provided that they master the system and know its particularities[9]. But these numbers in ancient writings do not leave the context of private or semi-public use, and are prohibited on identity cards, passports, checks, or again in scientific or technological articles. These pseudo-vestiges thus produced at present (for instance in the bicultural schools) do not allow a return without risk towards the past, and suffice only quite imperfectly in the quest for the lost numbers of the Mayas.

For spoken numerations, the current situation is a bit more nuanced, for it also depends on the linguistic resistance of the Mesoamericans, varying according to the peoples, the circumstances and the period. Thus, we observe a continuum of multilingual situations which simultaneously confronts those who only (or only still) speak Spanish with those who speak only one or more Indian languages. In any case, an indigenous speaker may know all or part of the spoken numeration of the Indian languages which have evolved[10]and which are themselves more or less open to borrowing and to mixing. The most frequent case at present is to hear mixes in which small numbers are most often spoken in the Indian language and the greater (especially when they relate to the globalized world) in Spanish. One reads, for example, in a text relating a marriage ceremony "tehuan quinequi ***dos ochenta*** uan ***chicuase*** totolme" or "we want **two hundred and eighty** pesos (in Spanish) and six **turkeys** (in Nahuatl)" (Dehouve;1978:190).

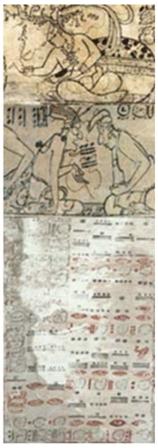
1.3 A brief survey

As they have disappeared, the modern readers can no longer debate face-to-face with the scribes of the classical period. In order to use the corpus of equations that utilize the lost numbers, the modern reader may, however, count on the collective capacity of the scholars who discover one by one the documents left by History, who decipher them and who translate them. It is through them that we can hope to enter into the cognitive universe – *a priori* strange and objectively foreign –: **the Mayan Arithmetic Intelligence**.

[8]It must be noted that the initial conditions of these two dramas were quite different. On one side, an Aztec Confederation with a centralized political system, hierarchical and strongly bonded by a very ritualized religion. On the other, dispersed Mayan populations among which one of the principal points in common was the continued divinatory use of the *tzolkin*, the "week or year" whose 260 dates allowed thousands of numerological inferences lending themselves to the acts of divination.

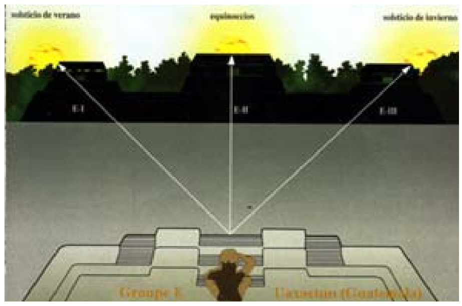
[9]For example, the best known irregularity is that of an 18-month year.

[10]The proactive formations characteristic of colonial Yucatec, are no longer used (and sometimes no longer even understood) by the speakers of the tens of Mayan languages still practiced.



Our advantage in this race for lost numbers is that our champion, the Mayan scribe of the Classical period, is without contest the only Mesoamerican prior to the meeting of the two worlds, who has left a treasure of mathematical vestiges. Sometimes this remained treasure is petrified in the ruins of astronomical observatories or in the alignments of monuments like the Temples of the group E of Uaxactun (Petén, Guatemala).

Sometimes in the mathematical tools painted in the codex in order to serve in the resolution of the problems of the mastering of time. The deciphering of the Dresden Codex confirms this observation and reveals certain instruments developed by the scribes: several systems of written numeration, several calendars, a forest of cycles, a system of units of measurement for time, multiplication tables for all of the cycles, tables of dates that are invariable by this or that multiple of these cycles.



In order to succeed in their activities, inextricably divinatory and astronomical, the Mayas needed writing (vase Kerr 1185), spaces for discussion and for calculation (vase Kerr 1196), and diverse arithmetical-calendarial tools such as tables of multiples and tables of dates (Dresden Code p. 24). They also needed astronomical observations. Before beginning, here are the principal tools of the Mayan astronomer/astrologist:

1) a spoken numeration of protractive-type
2) a logo-syllabographic writing system (Hoppan;2012)
3) a written positional and vigesimal numeration with zero that distinguishes between the cardinal and ordinal faces of the integers, and thus opposes the times and dates
4) a vigesimal system of units of time measurement (called the periods, $\mathbf{P_i}$): the **<u>tun</u>** "year" (360-d) plus the open list of its multiples like **<u>katun</u>** and **<u>baktun</u>**, and two submultiples (**<u>uinal</u>** "month of 20 days" and **<u>kin</u>** "day")
5) astronomical observatories
6) a graphic system which produced tables of multiples and charts of dates
7) **four intertwined calendars**, namely:
   a. the Long Count, **CL**, theoretically open and isomorphic to the set of natural numbers, which dates the day by their distance $\sum(\mathbf{c_i P_i})$ at the origin of Maya chronology (the 12/08/-3113 in concordance GMT with the constant 584 284)
   b. the *tzolkin* "divination week" of 13×20 days, which provides 260 dates $\boldsymbol{\alpha X}$
   c. the *ha'ab* "solar year" of 18 months of 20 days and 1 rest of 5 days, which provides 365 dates $\boldsymbol{\beta Y}$
   d. the Calendar Round, **CR**, which results from the combination of *tzolkin* and *ha'ab*, and which provides 18 980 dates of the form $(\boldsymbol{\alpha X}, \boldsymbol{\beta Y})$
8) ephemeris to track the phases of Moon and Venus, the return of eclipses⋯

1.4 Main thesis: Mayas and Aztecs had different manners of writing dates

Unlike the Mayas, the Aztecs did not have four intertwined calendars. They were using **two calendars**[11]: the first dates the days, and the second serves to identify the years. The first, called tonalpohualli in nahuatl, is the twin of Mayan tzolkin of 260 dates αX. The second is a cycle of 52 dates $\alpha \underline{X}_P$ that distinguish and define 52 years (forming what it is called the Aztec century, SA) through the intermediary of an agreement. One particular day, for instance the first day of the first month of the year, is distinguished and its tonalpohualli date $\alpha \underline{X}_P$ is used as the "proper name" of the year. In other words, that day is, by definition, the day **eponymous of the year**; it is also said the **Year Bearer**.

Only a detail separates Mayanists and Nahualists. For the former, the Maya calendar contains exactly 18 980[12] **kin** 'days'. For the latter, the Aztec calendar consists of 52 *xihuitl* years whose total in days – determined by the type of the year – is supposed to be the same: 52×365. A supposition to be confirmed or invalidated. Showing that the Maya CR of 18 980 days and the Aztec *xiuhtlalpilli* of 52 years are two different types[13]of calendars, this presentation invites the reader to reconsider the traditional thesis according to which the sharing of a mutual calendar is a defining trait in the concept of Mesoamerica (Kirchhoff, 1943).

All Mesoamerican calendars seem to be defined by a common structure of a product determined by cycles whose root is the combination of an almanac of divination of 13×20 days and a vague year of $(18 \times 20) + 5$ days[14]. The calendar expressions in fact combined in the Mesoamerican *hic et nunc* are, however, visibly different between the two peoples. Each city followed its own rules for dating events and distinguishing days.

Common to all Mesoamericans, the 'almanac' dates are confirmed[15]from the middle of the VIIth century B.C. until the colonial period. These are expressions of the form αX, where α follows a cycle of 13 successive wholes, and X a cycle immutably ordered of 20 day-glyphs. In other words, a cycle which is the product of 13 x 20 = 260 dates, established with the order: $s(\alpha X) = [s_1(\alpha), s_2(X)]$ where the 's' are the 'successor' functions of the cycles being considered.

[11]Like all peoples, the Aztecs have been subjected to the seasons, to the variations of the received solar energy. They divided the solar year into 18 months but they did not do an annual 365-calendar and they wrote no date of the form βY (Cauty:2012).

[12]GCM (260, 365). Is one fifth of the product $tzolkinxha'ab = 94900$. It is also 949 **uinal** 'months', 73 *tzolkin* or 52 *ha' ab*. Its double, 2 CR, equals 65 cycles of Venus.

[13]In spite of a common origin and important similarities such as those that can be made between the Mayan 52-cycle of the Bearers and the Aztecan 52-cycle of the Eponyms.

[14]That is to say: Mayan tzolkin or Aztec tonalpohualli of 260 days and Mayan ha' ab or Aztecan xihuitl of 365 days.

[15]The oldest are anthroponyms (Urcid, Pohl and others).

Nearly absent with the Aztecs, the vague year dates are later and are only clearly attested to with the Mayas[16]. These are of the form βY, wherein β follows the cycle $(0, 19)$ and Y the ordered cycle of 18 named[17]Months, *veintena*, of 20 days, and of the named compliment **Uayeb**. Thus a set of 365 days/dates for the vague year solar *ha*' *ab*, equipped with the order s' :

$$s'(\beta Y) = [s_3(\beta), Y] \text{ for each } Y \neq Uayeb \text{ and } \beta < 19$$
$$s'(19Y) = [s_3(19), s_4(Y)] = [0, s_4(Y)] \text{ for each } Y \neq Uayeb \text{ and } \beta = 19$$
$$s'(\beta Uayeb) = [s_3(\beta), Uayeb] \text{ for each } \beta < 4$$
$$s'(4Uayeb) = [s_3(4), s_4(Uayeb)] = 0Pop$$

2 The product $tzolkin \times ha'ab$ of the Maya and the cycle of Bearers

The CR[18]Maya is the ordered product $tzolkin \times ha'ab$ whose elements are couples $(\alpha X, \beta Y)$. The study of the mathematical properties of the ordered products of ordered cycles has shown (Cauty;2009:20-30) that the conjunction of three factors[19], along with the fact that 260 and 365 are multiples of the same number[20], has two consequences. First, to limit the number of CR dates to 18 980. And to produce, in addition, cyclic events[21]that are resumed by the **theorem**:

Whatever the integer P, the almanac date of the P^{th} day of the vague year is of the form αX_P, where α follows the cycle of 13 integer almanac dates, and where X_P follows a class, modulo 5, of four X day names.

Each day of the vague year is thus associated with $13 \times 4 = 52$ almanac dates who characterize it and succeed one another year after year according to the law: $s(\alpha X_P) = s(\alpha)s(X_P) = [(\alpha + 1), (X + 5)]$. The value $P = 0$ distinguishes and defines the 1st day of the 1st month of the Mayan year, the New Year. Applied to this day, the theorem states, first, that the Mayan New Year is associated with four[22]tzolkin X_P day names. And, additionally, that each New Year date αX_P distinguishes and defines a ha' ab year in the group of 52 years that make up the CR. In other terms, the system of dates αX_P supplied a practical means[23]to distinguish and name the years of a CR: by making αX_P the eponym for the year.

[16]The ***0 Yaxkin*** of the Leyden plaque (17/09/320 greg.) is among the first evidence of βY dates. The latter, from the Post Classic, are in the codex (Dresden and Paris).

[17]In colonial Yucatec, for example: Y covers the following (***Pop, Uo, Zip, Zodz, Tzec, Xul, Yaxkin, Mol, Ch' en, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, Cumku***) that closes the compliment ***Uayeb***. To know the date βY of a day is equivalent to knowing its position γ in the *ha' ab*: the cycle of the ha' ab dates ordered by s' is isomorphic to a group of 365 natural integers fitted with the natural order of integers.

[18]Its Mayan name is unknown. But its value of **2.12.; 13.0.** is established. Mayanists use the expressions: *Calendar Round, Wheel Calendar, Ritual Calendar.*

[19]F1: the rules for formulating the expression composed of dates (*tzolkin, ha' ab* and CR), F2: the type of enumeration of the pairs $\alpha X \beta Y$, and F3 : the relative position (determinable by the origin date **4 Ahau 8 Cumku**) of tzolkin and ha' ab at the moment of starting the CR.

[20]Their GCD 5, greatest common divisor.

[21]Occasion to celebrate, each year among the Maya, the change of year Bearer; and every 52 years among the Aztecs, the Binding of the years (*xiuhtlalpilli*) and the New Fire.

[22]The entities associated with these 4 names are called the 4 Year Bearers: **Ik, Manik**, Eb and **Caban** in classic Mayan.

[23]Probably not used in the classic period because the dating methods were particularly redundant. Especially in the solemn public use of steles and monuments recounting the glory of Mayan cities and leaders in the enameled texts of dates given in the CR system, but also in the Long Count $\sum c_i \underline{P}_i$, in Lunar Series, and other cycles as well. For example: CL **13-<u>baktun</u> 0-<u>katun</u> 0-<u>tun</u> 0-<u>uinal</u> 0-<u>kin</u>** and the date CR **4 Ahau 8 *Cumku*** from the Stele E of Quirigua (Guatemala, 771 A.D.).

3 *Tonalpohualli, xihuitl* and Aztec eponyms

Like all Mesoamericans, the people submitting to the Aztec Triple Alliance used the αX dates of the almanac. Like everyone else, the Aztecs underwent and observed diurnal and seasonal variations of the solar radiation and they used, as all Mesoamerican peoples, a year (not a calendar) we refer to as 'festive'. The festive year has 18 months of 20 days each and 1 residue of n days. Theoretically: each period has a proper name, the residue ***Nemontemi***[24]has 5 days, and all 19 periods are immutably ordered. So, we can refer to the festive year by the ordered list ***I***, ***II***, ***III***, etc., ***XIX*** of its periods.

But up to here, the pre-Colombian documents delivered **no Aztec date of the shape** βY distinguishing (as our January 1st or as the Mayan ***0 Pop***) the days of the solar year by their rank in their month Y. The fact is that: prior to the arrival of the Spanish, the Aztecs have not left written βY dates with the help of the glyphs of the indigenous pictographic writing. From the Spanish Conquest, Mesoamericans were all forced to follow the European calendar, a solar calendar divided in 12 months. At the same times, we observed a very few documents which transcribe into the Latin alphabet the detailed forms for a handful of event[25]dates that were critical for both Worlds.

In these conditions, it is possible to reconstruct an Aztec βY date[26]; and it is possible to reconstruct some complete expressions like "the day **8 Ehecatl** ***9 Quecholli*** of the year **1 Acatl**" for Cortez' entry into Mexico City. Usually not recorded, the rank within the month remains uncertain when we have it available. The reference ***9 Quecholli*** for example do not allow confirmation that the day corresponding to this date was, indeed, at the position ***9***, ***1***, ***10*** or ***20*** of the month Y. Why is this ?

Because the sources do not say how the Mesoamericans enumerated and counted their days. We don' t know for instance if they all began with the same number. However, we are certain that the ways of counting were diverse: the Mayas wrote the numbers from ***0*** to ***19***, the Spanish numbering the days from ***1*** to ***31*** and the Tlapanèques from ***2*** to ***14***.

From which are derived two deductions:

1) The CR of 18 980 days distinguished and dated by as many expressions $\alpha X \beta Y$ is not a tangible reality in the space/time of the Triple Alliance.
2) We can however, from a colonial date like ***9 Quecholli*** for example, undertake the reconstruction of the 365 dates βY of a xihuitl[27].

[24]Except for those who attribute to Mesoamericans the use of a 366-day leap year, sources state that the ***Nemontemi*** contained 5 unlucky days, unnamed, sleeping··· In *Historia general de las cosas de Nueva España*, Sahagún gives a list of 18 expressions traditionally accepted as that of the 18 month names although they are "very different from the point of view of their syntactical structure: 'there are gifts of flowers', 'the trees sit up straight', 'little watch', 'skinning people', etc." (Launey;2009:personal communication). Sahagún also gives the associated divinities, the name of the period *Nemontemi*, as well as the position of the period in the Julian calendar. The second *veintena*, ***Tlacaxipehualiztli***, went from March 4 to 23; dedicated to Xipe Totec, it was characterized by the skinning of people. The list is attested to by multiple sources, *modulo* differences: the number of days and the position in the year of the period *Nemontemi*, the month that begins the year and subsequently the numeration of the months. The month ***Quecholli***, for example, is generally the 14th month, but it is the 13th for Rámirez. In two texts (manuscript 215 and *Historia antigua de Mexico*) Ixtlilxochitl begins/ends the year in: ***Atlcahuallo/Nemontemi*** and ***Atemoztli/Panquetzalztli*** (Roulet; 1999).

[25]The most credible eyewitness reports concern Cortez' entrance into Mexico City (08/11/1519), the Night of Sorrows when he is driven out, and the destruction of the Temple of Mexico (Tena:1987;ch. IV).

[26]To which we may add more vague indications stating, for example, that the twenty monthly ceremonies took place at the beginning, the middle or at the end of the *veintena*. Usually not recorded, the rank within the month remains uncertain when we have it available.

[27]In a more or less credible manner according to the suppositions retained, beginning with that of the number of days attributed to the *xihuitl* (365 or 366?). Or to reconstruct the βY dates of the 52 years of a *xiuhtlalpilli*, but this with yet more uncertainty.

Besides the formula of xihuitl and its invisible dates[28], the Aztecs inherited, on one hand, knowledge of the duration (52, in number of years) of the CR and, on the other hand, of the effects of the conventions that structured it and which began the Bearer cycle. This heritage takes into account the representations of successions of xihuitl and most importantly the habit of distinguishing and noting the years by the ordered succession of their eponyms $\boldsymbol{\alpha \underline{X}_P}$[29]like that of the folio 2r of Mendoza going from the year **2 Calli** to the year **13 Acatl** in passing by **2 Acatl** signaled as the year of the celebration of the New fire. Thus, as we had seen (date on the Dedication Stone of the Templo Mayor), the Aztec date for the day of an event is an expression $(\boldsymbol{\alpha X}, \boldsymbol{\alpha \underline{X}_P})$, where $\boldsymbol{\alpha X}$ is the tonalpohualli date of the day of the event and XP that of the eponym day of the year in which it occurs. This mode of dating does not result in 18 980 dates as in the case of the Mayan CR, but only 260 x 52 = 13 520 possible different expressions[30]. It is, by its construction, ambiguous: 260 dates $\boldsymbol{\alpha X}$ are not sufficient to distinguish the 365 days[31] of a xihuitl; and 13 520 pairs $(\boldsymbol{\alpha X}, \boldsymbol{\alpha \underline{X}_P})$ are not sufficient to date the days of an Aztec Century of 18 993 days – in verdadero (Sahagún) and Julian calendars – or of 18 980 days if one decides to identify it[32] with the Mayan CR.

4 Difficulties and differences

Identifying Mayan and Aztec calendars is a habit[33]whose principal fault is to hide a specific difference: only the Classical Mayas commonly wrote $\boldsymbol{\beta Y}$ dates of the annual *ha' ab* calendar. When it is noticed[34], the difference – the inscription of the eponym *vs.* the *ha' ab* date – is sometimes denied by an interpretation that reduces it to a question of abbreviation or preference[35]. But no: small cause, big consequence. This "preference" would give18 980 dates $(\boldsymbol{\alpha X}, \boldsymbol{\beta Y})$ to the Mayas; and 13 520 dates $(\boldsymbol{\alpha X}, \boldsymbol{\alpha \underline{X}_P})$ to the Nahuas. This is not a stylistic nuance. The Aztecan expression "8 ehécatl of 1 ácatl" and the Mayan "4 aháu 8 cumkú" are fairly different; and they are not, as Tena (2000) wrote, two "abbreviations" which denote similar or identical conceptualizations of the same "fact

[28]The bY dates of type 9 Quecholli that we never see written in indigenous glyphs.

[29]$\boldsymbol{X_P}$ are the 4 days, linked to the Bearers among the Mayas. Among the Aztecs, these are the days: **Calli, Tochtli, Acatl** and **Tecpatl** whose Mayan equivalents are **Akbal, Lamat, Ben** and **Edznab**, witnessed in the Dresden Codex, but which are neither **Ik, Manik, Eb** and **Caban** presented in Classical Mayan nor **Kan, Muluc, Hix** and **Cauac** recorded in the Colonial Period by Landa and the Madrid Codex.

[30]Not a proper part of the product, though 52 and 260 are divisible by 4 (see the CR), because the law of succession ('linear enumeration', type: 1 January, 2 January, etc.) defined in *tonalpohualli x xiuhpohualli* – $\boldsymbol{s(aX, aXP) = [s(aX), aXP]}$ as long as the xihuitl is not passed, otherwise $\boldsymbol{s(aX, aXP) = [s(aX), aXP + 1]}$ – is not the same as that defined in $tzolkin \times ha'ab$ ('diagonal enumeration', type: Monday 1, Tuesday 2, etc.).

[31]It is always possible to render the calendar unambiguous by taking an additional cycle, for example, that of the 9 Lords of the Night.

[32]Decision leading to the creation of 52×105 double dates $\boldsymbol{\alpha X}$ to add, in an order to be specified, to the 260 already used in each *xihuitl* year of the Aztec century.

[33]"El calendario mesoamericano era el resultado de la combinación entre un ciclo de 365 días, llamado en náhuatl *xiuhpohualli* o "cuenta del año" (*ha' ab* en maya), y otro ciclo de 260 días, llamado en náhuatl *tonalpohualli* o "cuenta de los días" (*tzolkin* en Maya) [⋯] Se requería el transcurso de 18 980 días nominales, equivalentes a un "siglo" de 52 anos, para que se agotaran todas las posiciones posibles de un día cualquiera del *tonalpohualli* dentro del *xiuhpohualli*, y viceversa" (Tena;2000:5).

[34]Which it is not always, because the habit of transcribing everything (*tzolkin* date, *ha' ab* date, period, etc.) in the same manner neutralizes and renders invisible many of the differences noted by the scribes.

[35]"Tanto los nahuas como los mayas utilizaban una fórmula **abreviada** para los fechamientos, pues ordinariamente no se mencionaban en forma completa todos los elementos que intervenían en una fecha, a saber: el día del *tonalpohualli*, el ordinal del día dentro de la veintena, y el año. Los nahuas **preferían** enunciar sólo el día del *tonalpohualli* y el año; decían, por ejemplo, *8 ehécatl* de *1 ácatl*. Los mayas, en cambio, sólo enunciaban el día del *tzolkín* y el ordinal de la veintena; decían por ejemplo: *4 aháu 8 cumkú*." (Tena; 2000: 5. AC underlined in bold).

of calendar". These are two types of dates to be analyzed //**8 Ehecatl/1 Acatl**// and //**4 Ahau/*8 Cumku***//, of very different components. The 1rst does not introduce any difference: for both an Aztec and a Maya, it is a $\boldsymbol{\alpha X}$ almanac date. But the 2nd component is, both in nature and form, quite different. In Nahua: another date $\boldsymbol{\alpha X_P}$, but in Mayan: a date $\boldsymbol{\beta Y}$. There is no isomorphism[36]. But rather a Type B elder who led the Classical Mayas to write dates $(\boldsymbol{\alpha X}, \boldsymbol{\beta Y})$ and to imply the eponym $[\boldsymbol{\alpha X_P}]$; and a Type A who, during the Postclassical, led people to write dates $(\boldsymbol{\alpha X}, \boldsymbol{\alpha \underline{X}_P})$ and to imply the date $[\boldsymbol{\beta Y}]$. The formulas are:

$$\text{Type } B = (\boldsymbol{\alpha X}, \boldsymbol{\beta Y}), [\boldsymbol{\alpha X_P}] \text{ among the Mayas}^{37}$$
$$\text{Type } A = (\boldsymbol{\alpha X}, \boldsymbol{\beta Y}), [\boldsymbol{\beta Y}] \text{ among the Aztecs}^{38}.$$

5 Reflections and conclusions

The non-isomorphism of the Aztec and Mayan calendars possibly has its root in the fact that only the Mayas used the long Count $\sum \boldsymbol{c_i}(\underline{\boldsymbol{P_i}})$ and the CR $(\boldsymbol{\alpha X}, \boldsymbol{\beta Y})$ in a joint and immutable manner. This usage led, in effect, to the rigorous synchronous maintenance of the tzolkin and ha' ab cycles and to never change the number of days of the vague year or of any other unit. The real benefit of this rigor was, of course, the possibility of making precise calendar calculations[39] with the help of the Multiplication Tables[40] and the Date Tables that we find so abundantly in the Codex and which served to accomplish the challenges of modular Mayan arithmetic[41].

Without this rigor and these tools, the scribes would undoubtedly not have been able to simulate the return of eclipses, correct the shortcoming of the Venusians leap year or record the embellished narrative tales of a network of dates and numbers of distance that celebrated the grandeur of gods, cities, kings.... Otherwise stated, it is indeed the rule ROCm (Cauty; 2009b:10-12), consequence of the circumstances described above, and cause of the specificities expressed in § 2 by way of the **theorem** which serves as the basis of the possibility of calculating to the day, and which puts it into practice with the constraints of co-occurrence imposed on couples of the product $tzolkin \times ha'ab$. It will be further shown that the redundancy brought by the use of the CL[42]acted as a code detecting, even correcting, errors.

[36] In consequence of this non-isomorphism: the contrast between the Mayan facility to find the eponym $\boldsymbol{\alpha X_P}$ from the date $\boldsymbol{\alpha X \beta Y}$ of an event, and the difficulty for a Nahualist to find the date $\boldsymbol{\beta Y}$ from the date $(\boldsymbol{\alpha X}, \boldsymbol{\alpha X_P})$. A Mayan date $\boldsymbol{\alpha X \beta Y}$ defines one and only one day of the CR. The position in the ha' ab of the day $\boldsymbol{\beta Y}$ is "$\boldsymbol{\beta \to Y}$" in spoken protractive numeration. Subsequently, the eponym (*tzolkin* date of the day ***0 Pop***) sought is given by $\boldsymbol{\alpha X_P} = \boldsymbol{\alpha X} - (\boldsymbol{\beta \to Y})$. For example: **7 Eb** is an eponym for the year of **4 Ahau 8 Cumku**. Another facilitating factor for the Mayanists is the richness of redundant elements in the calendar expressions.

[37] Whose calendar system imposed the constraint of co-occurrence on the marked components (*tzolkin* date and *ha' ab* date) and implying the redundant eponym.

[38] Whose calendar system imposed writing the eponym $\boldsymbol{\alpha X_P}$ of the year, but no constraint of co-occurrence on the *tonalpohualli* dates (marked) or the *xihuitl* dates (never written).

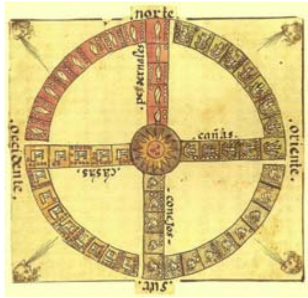
[39] In practice, to 1 day. The fact that the scribes used all sorts of cycles may be interpreted by saying that they calculated in rings Z/nZ or in classes of appropriate integers modulo n. Clocks give us a familiar image of this type of calculation which lowers by n any number that reached or surpassed this value, because the hour hand makes additions modulo $n = 12$: if it starts from 7 o'clock, for example, and ten hours must be added, it will not mark 17:00 hours, but 5 o'clock.

[40] Containing at times, in the position of intruder, non-multiple numbers serving to correct the calendar deviation in vague years, like that of Venus of the Dresden Codex.

[41] Given 2 dates $\boldsymbol{x}$, $\boldsymbol{y}$ (of the Mayan CR to clarity things) and one translation $\boldsymbol{t}$ (in whole number of days), solves the 3 equations $\boldsymbol{t}(\boldsymbol{x}) = \boldsymbol{y}$ according to whether the unknown is $\boldsymbol{x}$, $\boldsymbol{y}$ or $\boldsymbol{t}$.

[42] But also the Numbers of distance, lunar Series, and other cycles, like that of the Lords of the Night (or patrons of the Otherworlds).

For the Aztecs, the product $(\boldsymbol{\alpha X}, \boldsymbol{\alpha \underline{X}_P})$ of the dates *tonalpohualli* × *xiuhpohualli* is not limited by the factors F_1, F_2 and F_3 because the $\boldsymbol{\beta Y}$ dates were no longer written. Also, in the absence of the Long Count[43], eventual discrepancies or other calculation errors become readily unapparent. The Aztec century, *xiuhtlalpilli*, was thus freed from the functional obligations which the ROCm Rule imposed upon the product *tzolkin* × *ha'ab*. Due to this, the cycles $\boldsymbol{\alpha X}$ and $\boldsymbol{\alpha X_P}$ were like "free spinning gears" in relation to one another. The Almanac had sorted out its 260 $\boldsymbol{\alpha X}$ dates, but it needed something or someone to increment and count the eponyms $\boldsymbol{\alpha X_P}$ and to maintain the 52-year cycle no longer rigidly tied to the *xihuitl*.



Colonial or not, the sources do not explain how Mesoamericans without $\boldsymbol{\beta Y}$ dates knew when to increment the eponym. In principle an Aztec was not obliged to change *xihuitl* like a Maya would change *ha' ab*, that is to say: on the passage from the 365th and last day of the year $n-1$ dated ***4 Uayeb*** to the 1rst day of the year n dated ***CHUM/0 Pop***[44]. Numerous figures revealing the persistence of the number and of the order of succession of the 52 years of a *xiuhtlalpilli* prove that the tradition of the vague year of 365 days was maintained (Durán Codex).

However, the insistent indication – stating, without proof, that the Natives corrected the vague year, that they possessed a "real" (*verdadeiro*) calendar to which they added a 366th day every four years – proves two things relative to the Postclassic and Colonial calendar customs. Firstly, that they knew the vague year of 365 days. Secondly, that it seemed to remain in phase with a tropical year of 365.25 days or with a vague year of variable length[45].

Of course, the Aztecs could recognize the changing of seasons or of years. A reasonable hypothesis is thus to suppose that the changing of a year and of the eponym $\boldsymbol{\alpha X_P}$ could be decided according to the appearance of a designated natural or astronomic sign: the passage of the Sun at the Zenith of a site[46], a Solstice, a passage at the meridian of *Miec/Tianquitzli* (Pléiades), the Bridge of Turtles...

In *Los observatorios subterráneos*[47], Rubén B. Morante López evaluates the research and the measures taken since the eighties by Aveni and Hartung (1981), Anderson (1981), Broda (1986, 1991) and Tichy (1980, 1992) in the subterranean observatories, notably that of Xochicalco[48]. The authors recorded the days upon which rays of light enter into the chamber of the observatory, and the days upon which they do not. The results show that those who conceived the observatories constructed

[43]The "small" generative capacity of the Aztec written numeration and its repetitive form are elements unfavorable to the writing of large numbers and the composition of tables of multiples, that partially explains this absence of Aztec Long Counts.

[44]Which amounts to changing the manner of counting the days, following the end of the installation of the New Year. Between ***0*** and ***1 Pop*** or between ***1*** and ***2 Pop*** or ***2*** and ***3 Pop*** according to the strategies of enumeration of the ranks $\boldsymbol{\beta}$ and the counting of days that we know to have started at ***0***, ***1*** or ***2*** according to the peoples and the periods.

[45]Consequently, the dates (eponym, n^{th} day of the month, New Year, etc.) do not stray in the seasons like the Mayan ***0 Pop*** should – in spite of the affirmations by Landa who fixed the Mayan New Year on the Christian July 16th, and asserts that the scribes added a 366th day to the year every four years.

[46]In the intertropical zone, the Sun passes twice at the Zenith (before and after the Summer Solstice), and each passage is easily noted by the absence of shadow for objects held vertically (steles, for example). One may also follow the rising and setting of the Sun in relation to reference points of the horizon or of the city, etc. All of this provides the means to elaborate an annual calendar in phase with the tropical year.

[47]On-line: `http://www.uv.mx/dgbuv/bd/pyh/1995/2/html/pag/index.htm`, the article appeared in 1995 in *Lapalabra y el hombre*.

[48]Are noted (Morante Lopez;2001:48) the subterranean observatories of Teotihuacan (200 AD), Monte Alban (400 AD) or Xochicalco (700 AD).

the chimney in such a way as to divide the year in two: one part during which the chamber received the Sun's rays, and a part in which it did not. According to the measurements taken in 1988 – 1992, the first period began on April 30 (once on May 1) and the second on August 13. These dates divide the year into a portion of 105 days and one of 260 days (261 in leap years). The 105 day portion is centered on the Solstice[49] of June 21: April 30 + 105 = August 13, August 13 + 260 = April 30[50], April 30 + 52 = June 21, June 22 + 52 = August 13.

The Aztecs thus disposed of a sort of clock or of calendar giving, live and continuously, the progression of the days and seasons of the tropical year:

	Solstice			
01/05	21/06	12/08	August 13	April 30

The interest of the experiment of Xochicalco is to reveal the following points:

Certain Mesoamericans constructed heliographs (markers of rays of light) that gave the beginning and end of two periods.

A lighted period that includes the principal bearings of the tropical year (Summer Solstice and passages at the Zenith) and whose length of 105 days enjoyed undeniable numerological properties. For example: 105 = 5 x 20 + 5 = 2 x 4 x 13 + 1.

A period of shadow lasting 260 days equaled the length of the divinatory almanac and catches up with, in four years, the delay of one day that the vague year has on the tropical year. The period of shadow lasts 260 days in a normal year and 261 days[51] in leap years.

Vague Year No. 1		Vague Year No. 2		Vague Year No. 3	
105 days	260 days	105 days	260 days	105 days	260 days
Vague Year No. 4		**Vague Year No. 5**		**Vague Year No. 6**	
105 days	**261 days**	105 days	260 days	105 days	260 days

Disposing of such a heliograph, the kings and priests have no need of a calendar of the vague year, nor even to mark the 365 passing days. Because, in order to know at which point was the tropical year, it was sufficient to go to the observatory to read and interpret what the Sun's rays revealed in this sacred place. And to decide, for example, to increment the Book of Years or to celebrate the New Fire. Without possible error. But in a manner quite distant from the calendar and computational habits of the Classical Mayas[52].

But not everyone disposes of an underground observatory. During the Postclassic and Colonial period, most of the peoples contented themselves with the dates $(\alpha X, \alpha \underline{X}_P)$ and to follow in parallel the course of the months of year and the rhythm of the 20 monthly celebrations. In the areas mixed by contact with the Spanish, the Natives had every interest in hiding their attachment to the sacred

[49] Framed by the 2 passages at the Zenith whose date depends on the latitude of the site.
[50] August 13 + 261 = April 30, during leap years.
[51] Because d(August 13, April 30/May 1) = 260/261 according to whether February counts 28/29 days.
[52] At best linked by a cryptomorphism the calendars $(\alpha X, \beta Y)$ and $(\alpha X, \alpha X_P)$ do not even speak the same language. The Aztec Century is an adjustable simulation of the solar year while the Mayan CR is an untouchable arithmetic model made to distinguish and to define each of the 18 980 days of the most typical Mesoamerican temporal cycle. At the cost of losing dates during seasons without making any claims for a "true calendar"

almanac and to the eponym cycle, largely stigmatized as Satanic works. One perceived way to do so consist of becoming a user of the vague solar year calendar. *Ha' ab* and *xihuitl* are, indeed, much close to the calendar of the Spaniards, even if they contain 18 months of 20 days instead of 12 months of 30 days on average. Or, the Colonial sources contain Indian calendars that seem to stem from this state of things. These are the annual calendars that respond like the Maya *ha' ab* or the Aztec *xihuitl* to the formula (18 x 20) + 5. But, that differ from it in the manner of writing the 20 dates of a $\boldsymbol{Y}$ month. At a first analysis, these calendars reveal three types of different situations:

1) The dates of the Mayan months of the Classic Period are of the form $\boldsymbol{\beta Y}$ and, without going into the details of the representation of ***Uayeb***, we may represent the *ha' ab* calendar by a table of columns labeled by the 18$\boldsymbol{Y}$ names[53] of the moths and learned by the sequence of the twenty first natural integers which semiotize the twenty $\boldsymbol{\beta}$ of the days in the month[54].
2) For the Mayas of the Postclassic or Colonial periods, the months remained unchanged and continued to label the columns of the chart, but the $\boldsymbol{\beta}$ positions are no longer written[55]. The monthly columns are, however, learned by the sequence of the 20 $\boldsymbol{\alpha X}$ dates of the days of the month, to which the rows $\boldsymbol{\alpha}$ bring a strong character of *trecena*. More precisely, the columns are indicated by the dates $\boldsymbol{\alpha_i X_j}$ where the references i and j vary from 1 to 20, modulo 13 (for the $\boldsymbol{\alpha}$ rows) and modulo 20 (for the $\boldsymbol{X}$ names)[56]. In the example of the *Calendario de los Indios de Guatemala*, the 20 lines of the days of a month are additionally numbered from 1 to 20 and specifying that it consists of the position of the day (Día) in the month of twenty days. This is evidently a notation made by/for a Spaniard, and not to assign to the $\boldsymbol{\beta}$ rows of the days in the Mayan month of the Classic Period (whose variable interval ranged from ***0*** to ***19***).
3) For the non-Mayas of the Post Classic or Colonial Periods, the months are not identified by their $\boldsymbol{Y}$ name[57], but by a scene or a description which seems to vary locally. A bit like the how expression *Le temps des cerises* allows a French person to identify the month of June[58]As in 2), the columns of months are indicated by the dates (*tonalpohualli*) of the form $\boldsymbol{\alpha_i X_j}$.

As shown by the examples of Annexes, it results from 1) and 2) that the Mayan *ha' ab* remained isomorphic, which amounts to saying that the ROCm rule remained in application. No longer recording the $\boldsymbol{\beta Y}$ dates allowed, however, the tolerance of minor deviations: to change the initial positions of the *tzolkin* and *ha' ab* cycles in relation to one another at the starting moment of the CR. One such deviation reveals itself by a change in the role of the Bearers. But it does not modify the organization of the CR, which remains a calendar of 18 980 dates. In this case, it would be legitimate to reconstruct, from a fragment, the 365 dates of the annual calendar, or the 18 980 of a CR.

In the cases 2) and 3), taking into account the bias introduced by the Colonizer/ Colonized contacts, and the imprecision already signaled concerning the eventual length of the *xihuitl* (365? 366?

[53]The $\boldsymbol{Y}$ names of the months evidently change with the languages and the periods, but all of the lists contain, with the exception of one translation, the same months in the same order of the type ***Pop, Uo***, etc., ***Cumku, Uayeb***.

[54]These 20 rows vary from **20/0** (***TI' HA' B/CHUM***) to **19**.

[55]Sometimes doubled by the number y of its position in the succession of the months of the *ha' ab*. Sometimes translated into Nahuatl or another language, and sometimes doubled by a description in Spanish or Latin. For example, in the *Calendario de los Indios de Guatemala, 1685, Cakchiquel*, `http://famsi.org/research/mltdp/item57` we have: *Mes n° 10 Rucactoeie.*

[56]Reading horizontally, the Xj are constant, and the ai in arithmetic progression with a common difference of 7 (modulo 13), so that, in 1 out of 2 columns, they are in the natural order of integers.

[57]By a glyph of the month, or by the $\boldsymbol{y}$ position of the month in the succession of the 18 months of a year.

[58]To cite an example: `http://www.lunacommons.org/luna/servlet/view/all/who/Tovar`.

365.25?), the dating $(\boldsymbol{\alpha X}, \boldsymbol{\alpha \underline{X}_P})$ does not, on its own, allow the reconstruction, to the day, of the 18 980 dates of a Mayan CR cycle[59]or the dates of the days of an Aztec Century of 52 years. From there results the question of a possible corrective[60]of the delay of the *ha' ab* calendar or *xihuitl* over the tropical year.

Revisiting the Mesoamerican calendars would consist of crossing a typology of the calendars[61] with a typology of the situations and users[62] .

This work could cast new light upon the diversity of the roles of the Bearers, eponyms and calendars, and show that the kings of cities with heliograph could have, without writing $\boldsymbol{\beta Y}$ dates, maintained in phase the succession of the *xihuitl* and that of the tropical years. Otherwise stated, here or there, the *Calendario verdadero* may well have had a pre-Hispanic reality. Certainly, different both from the reality of Julian and Gregorian calendars and that of reinterpreted Mesoamerican calendars, in the *mestizo* zones of interaction between the two Worlds, by and for the Colonial and Evangelist institution of the Europeans.

REFERENCES

- Alva Ixtlilxochitl, F, 1977, in *Obras Históricas*, Tomo 2, O' Gorman (eds.), México, UNAM, Instituto de Investigaciones Históricas, p. 565.
- Aveni, A., 1981, "Tropical Astronomy", in *Science*, Vol. 213, n° 4504, p. 161-171.
- Cauty, A., 2009a, *Des Porteurs mayas aux éponymes aztèques*, `http://celia.cnrs.fr/FichExt/Etudes/Maya/Cauty_DatesAnneeVague.pdf`
- Cauty, A., 2009b, *Y a-t-il des années surnuméraires mayas ?* `http://celia.cnrs.fr/FichExt/Etudes/Maya/CautyAnsMayasSurnumeraires.pdf`
- Cauty, A., 2012, *Xihuitl, l' année vague solaire vue par Durán*, `http://celia.cnrs.fr/FichExt/Etudes/Maya/Xihuitl_Duran.pdf`
- Edmonson, M., 2000, "Los calendarios de la Conquista", in *Arqueología mexicana*, vol. VII, n° 41, Mexique, p. 4045.
- Hernández Nieto, H., 1978, "Una interpretación diversa de la aritmética nahualt según un manuscrito de Juan Caramuel", in *Journal de la Société des Américanistes*, Tome LXV, Paris, pp. 87-101.
- Hoppan, J.-M., 2012, *Parlons maya classique*, Paris, L' Harmattan, à paraître.
- Kirchhoff, P., 1943, "Mesoamérica. Sus Límites Geográficos, Composición Étnica y Caracteres Culturales' ", in *Acta Americana* **1** (1), p. 92–107.
- Luria, A, 1966, *Les fonctions corticales supérieures de l' homme*, Paris : PUF.

[59]Marked by 52 $\boldsymbol{\alpha X_P}$ dates in the p. 34-37 of Madrid (dates $\boldsymbol{X_P}$ in p. 54-57 of the Dresden).

[60]Because correcting the vague year amounts to increasing the duration of a period and redistributing the normal/increased periods. This provokes a substantial delay likely to explain, for example, the diversity of the roles of the year Bearers or the dates $(\boldsymbol{\alpha X}, [\boldsymbol{\beta}]\boldsymbol{Y})$ of the Chilam Balam de Tizimin which contradicts the ROCm of the Classic.

[61]On the basis of respect of the ROCm or of the equation $x–\boldsymbol{\beta} = C$ of the deviations of these rules, we can distinguish, for example: calendars of the type Aztec Century or *calendario verdadero* of 18 993 days (52 *xihuitl* of 365/366 days), calendars of the type CR Mayan Classic of 18 980 days or its four clones, characterized by the roles P_0, P_1, P_2, P_3 and P_4 of the Year Bearers, even the total of 94 900 $(\boldsymbol{\alpha X}, \boldsymbol{\beta Y})$ possible pairs. A deviation from the ROCm rule may be provoked by a change in the variation interval of the rows $\boldsymbol{\beta}$, a change of synchronization $tzolkin \times ha'ab$ leading to start **Ahau (Chicchan, Oc, Men)** out of $\mathbf{0(5, 10, 15), 1(6, 11, 16), 2(7, 12, 17), 3(8, 13, 18)}$ or $\mathbf{4(9, 14, 19)}$, or a change in the duration of a period (day, month, year, century···.) to correct the delay of the vague year over the tropical year.

[62]For example: 'experts' (i) Mayan or (ii) Aztec *vs.* 'amateurs' (iii) Mesoamerican, *Mestizo* or Spaniard, of the Post Classic and Colonial Periods, etc.

DATES αX OF THE 20 DAYS OF AN AZTEC MONTH (*TECUILHUITL*, TOVAR CODEX)

4 Cipactli	**1 Ozomatli**
5 Ehecatl	**2 Malinalli**
6 Calli	**3 Acatl**
7 Cuetzpalin	**4 Ocelotl**
8 Cohuatl	**5 Cuauhtli**
9 Miquiztli	**6 Cozcacuauhtli**
10 Mazatl	**7 Olin**
11 Tochtli	**8 Tecpatl**
12 Atl	**9 Quiyahuitl**
13 Itzcuintli	**10 Xochitl**

– Marcus, J., 2000, "Los calendarios prehispánicos", in *Arqueología mexicana*, vol. VII, n° 41, Mexique, p. 12-19.

– Morante Lopez, Ruben B., 2001, "Las cámaras astronómicas subterráneas", in *Arqueología mexicana*, vol. VIII, n° 47, Mexique, p. 46-51.

– Polh, M., Pope, K., Nagy, C. Von, 2002, "Olmec Origins of Mesoamerican Writing", in *Science* 6, Vol. 298. no. 5600, pp. 1984 – 1987.

– Queixalos, F., 1986, "Autobiographie d'une néonumération", in *Amerindia*, 11, Paris : Association d'Ethnolinguistique Amérindienne, p.155-164.

– Roulet, E., 1999, "Une page curieuse sur les fêtes aztèques dans un manuscrit du XVIII[e] siècle", in *Journal de la Société des Américanistes*, 85, p. 367-373.

– Tena, R., 1987, *El calendario mexica y la cronografía*, México: Instituto Nacional de Antropología e Historia.

– Tena, R., 2000, "El calendario mesoamericano", in *Arqueología mexicana*, Vol. VII, n° 41, p.4-11.

– Urcid Serrrano Javier, 2001, "Zapotec hieroglyphic writing", in *Studies in Pre-Columbian Art and Archaeology*, n° 34, Dumbarton Oaks Research Library and Collection, Washington, D.C., 487 p

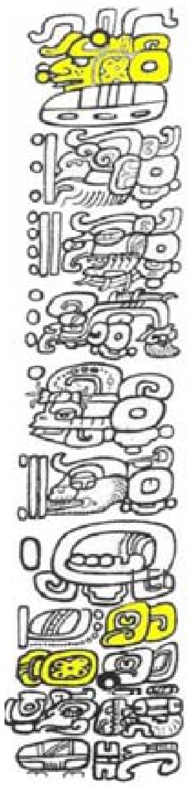
ANNEXES

The Mesoamerican year was a vague solar year composed of 19 periods/ months of 20 days, and a remainder of several days. According to the sources, the location, or the period, the remainder numbers 5 or 6 days. And there are two distinct ways to individually determine the days of the year. Like the Classical' Mayas, sequencing them by their rank β in the Y period, or like the Aztecs, distinguishing them by their date αX in the tonalpohualli (and year αX_P).

DATES $\boldsymbol{\alpha X_P}$ (EPONYMS) OF THE 52 YEARS OF THE AZTEC *XIUHTLALPILLI*

X_P = (Tochtli, Acatl, Tecpatl, Calli) codex Borbonicus, 19/21-20/22

THE 365 DATES βY OF THE MAYAN YEAR, HA' AB



8.14.3.1.12.
1 Eb ***0 Yaxkin***
(17/09/320)

Pop	*Uo*	*Zip*	*Zodz*	*Tzec*	*Xul*	*Yaxkin*	*Mol*	*Ch'en*	*Yax*	*Zac*	*Ceh*	*Mac*	*Kankin*	*Muan*	*Pax*	*Kayab*	*Cumku*	*Uayeb*
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	
12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	
13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	
17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	
18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	
19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	

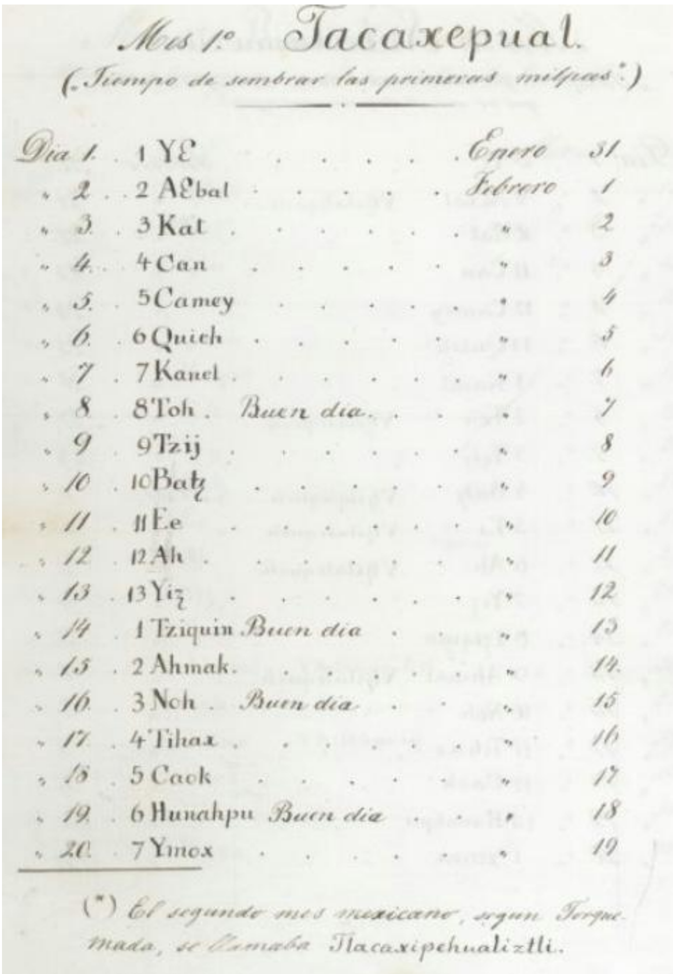
***0 Pop**, **1 Pop**, **2 Pop**, etc. **19 Pop**, **0 Uo**, **1 Uo**, etc., **4 Uayeb**.*

THE 260 + 105 DATES αX OF THE 365 DAYS OF A 19 PERIODS MAYAN *HA' AB*

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX
		Pop	*Uo*	*Zip*	*Zodz*	*Tzec*	*Xul*	*Yaxkin*	*Mol*	*Ch'en*	*Yax*	*Zac*	*Ceh*	*Mac*	*Kankin*	*Muan*	*Pax*	*Kayab*	*Cumku*	*Uayeb*
XX	Ahau	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9
I	Imix	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10
II	Ik	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11
III	Akbal	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12
IV	Kan	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13
V	Chicchan	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	
VI	Cimi	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	
VII	Manik	7	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9	
VIII	Lamat	8	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10	
IX	Muluc	9	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11	
X	Oc	10	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12	
XI	Chuen	11	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13	
XII	Eb	12	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	1	
XIII	Ben	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	2	
XIV	Hix	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9	3	
XV	Men	2	9	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10	4	
XVI	Cib	3	10	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11	5	
XVII	Caban	4	11	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12	6	
XVIII	Edznab	5	12	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	
XIX	Cauac	6	13	7	1	8	2	9	3	10	4	11	5	12	6	13	7	1	8	

13 Ahau, 1 Imix, 2 Ik, 3 Akbal, etc. 13 Ben, 1 Hix, etc., 6 Cauac, 7 Ahau, etc., 13 Cimi, etc., etc. 13 Kan.

DATES αX OF THE 20 DAYS OF A 1ST MONTH OF A MAYAN *HA' AB* (COLONIAL TIMES)



Mes 1º Tacaxepual.

(Tiempo de sembrar las primeras milpas.)*

Dia 1.	1 Yꜫ		*Enero*	*31*
2.	2 Aꜫbal		*Febrero*	*1*
3.	3 Kat		"	*2*
4.	4 Can		"	*3*
5.	5 Camey		"	*4*
6.	6 Quich		"	*5*
7	7 Kanel		"	*6*
8	8 Toh.	*Buen dia*	"	*7*
9	9 Tzij		"	*8*
10	10 Batz		"	*9*
11	11 Ee		"	*10*
12	12 Ah		"	*11*
13	13 Yiz		"	*12*
14	1 Tziquin	*Buen dia*	"	*13*
15	2 Ahmak.		"	*14.*
16.	3 Noh	*Buen dia*	"	*15*
17.	4 Tihax		"	*16*
18	5 Caok		"	*17*
19	6 Hunahpu	*Buen dia*	"	*18*
20	7 Ymox		"	*19*

(*) *El segundo mes mexicano, segun Torquemada, se llamaba Tlacaxipehualiztli.*

Calendario de los Indios de Guatemala, 1685, Cakchiquel
http://www.famsi.org/research/mltdp/item57/